## On the Bottom Magnetic Fields of the Millisecond Pulsars

Chengmin Zhang

School of Physics, The University of Sydney, NSW 2006, Australia

## Abstract.

The magnetic field strengths of most millisecond pulsars (MSP) are about  $10^{8-9}$  Gauss. The accretion induced magnetic field evolution scenario here concludes that the field decay is invesely related to the accreted mass and the minimum field or bottom field stops at about  $10^8$  Gauss if accreted with the Eddington accretion rate, which is proportionally related with the accretion rate as  $\dot{M}^{1/2}$ . The possibility of the low field  $\sim 10^7$  Gauss MSPs has been proposed for the future radio observation.

As known, the low magnetic field (MF) and fast spinning of MSPs are ascribed to the mass accretion in the binary system. The accretion induced neutron star magnetic field (NSMF) decay models have been proposed and paid much attention by a couple of authors (Romani 1990; Urpin & Geppert 1995; Cheng & Zhang 1998; Brown & Bildsten 1998; Payne & Melatos 2003), which is based on the observational evidence that NSMF decays in the binary accretion phase (Taam & van den Heuvel 1986; Shibazaki et al. 1989). Moreover, Van den Heuvel and Bitzaraki (1995) discovered that the increased amount of mass accreted leads to decay of the NSMF, and the 'bottom' field strength of about  $10^8$  Gauss is also implied.

Based on the accretion induced field decay in the slab geometry approximation proposed by Cheng & Zhang (1998), we extend it to the the spherical geometry structure of the accreted neutron star illustrated in Fig.1 of paper by Cheng & Zhang (1998). We assume that the magnetic field lines are frozened in the entire NS crust with homogenous average mass density. Under the condition of incompressible fluid approximation as well as the constant crustal volume assumption, the piled accreted materials will arise the expansion of the volume of the polar zone by  $\delta V_p = \frac{\dot{M}\delta t}{\rho} - A_p \delta H$ , where  $A_p$  is the surface area of the polar zone,  $\rho$  is the average density of the crust,  $\dot{M}$  is the accretion rate,  $\delta t$  a particular accretion duration, H the thickness of the crust and  $\delta H/\mathrm{H}$  the fraction of the thickness dissolving into the core which is defined by  $\frac{\delta H}{H} = \frac{\dot{M}\delta t}{M_{cr}}$ , where  $M_{cr} = 4\pi R^2 \rho H$  is the crust mass. The expansion of the polar zone will dilute the magnetic flux density if the conservation of the magnetic flux is preserved, so,  $\delta(BA_p) = 0$ , and the area  $A_p$  of the accretion polar patch can be accurately expressed as follows(Shapiro & Teukolsky 1983),  $A_p = 2\pi R^2(1-\cos\theta_c)$ ,  $\sin^2\theta_c = \frac{R}{R_A}$ , where  $\theta_c$  is the angle between field line of the star surface and polar axis,  $R_A$  is the Alfven radius,  $R_A = 3.2 \times 10^8 \, (cm) \, \dot{M}_{17}^{-2/7} \mu_{30}^{4/7} (\frac{M}{M_{\odot}})^{-1/7}$ , where M is the neutron star mass,  $\dot{M}_{17}$  is the accretion rate in units of  $10^{17} g/s$ , and  $\mu_{30}$  is the

magnetic moment in unit of  $10^{30}Gcm^3$ . Therefore, using the specified relation  $\delta V_p = H \delta A_p$  and the equation of conservation of magnetic flux, we get the following relation,  $\delta V_p = -V_p \frac{\delta B}{B}$ . Connecting the equations above, we obtain MF evolutionary equation as,  $A_p \delta B / [(2\pi R^2 - A_p)B] = \dot{M} \delta t / M_{cr}$ . Considering the initial condition  $B(t=0)=B_0$ , we have,

$$B/B_f = \{1 - [C\exp(-\frac{2\Delta M}{7M_{cr}}) - 1]^2\}^{-7/4}, C = 1 + \sqrt{1 - (\frac{B_f}{B_0})^{4/7}},$$

 $\Delta M = \dot{M}t$  and  $B_f$  is the magnetic field defined by the Alfven radius matching the star radius, i.e.,  $R_A(B_f) = R$ , which is also the minimum field strength, or

$$B_f = 1.32 \times 10^8 \left(\frac{\dot{M}}{\dot{M}_{Ed}}\right)^{1/2} \left(\frac{M}{M_{\odot}}\right)^{1/4} R_6^{-5/4} (G)$$

named as the bottom field (van den Heuvel & Bitzaraki 1995),  $B_f = 1.32 \times 10^8 (\frac{\dot{M}}{\dot{M}_{Ed}})^{1/2} (\frac{M}{M_{\odot}})^{1/4} R_6^{-5/4}$  (G), where  $\dot{M}_{Ed}$  is the Eddington accretion rate and  $R_6$  is NS radius in units of  $10^6$  cm. If  $\Delta M = 10^{-5} \sim 10^{-4} M_{\odot}$ , MF evolutionary equation can be simplified as the following approximated form,  $B = B_0/(1 + \frac{\Delta M}{m_B})$ , where  $m_B = \frac{1.32 \times 10^{-2} M_{\odot}}{1.02 \times 10^{-4} M_{\odot}}$  $\frac{1}{2}(B_f/B_0)^{4/7}M_{cr} \simeq 5 \times 10^{-4}(M_{\odot})(M_{cr}/0.2M_{\odot})$ . This is just the same form as the empirical formula of accretion induced field decay proposed by Shibazaki et al. (1989).

The main conclusions are summaried in the following. NSMF decays in the binary accretion phase, which is inversely correlated with the mass acreted from the companion. The bottom field of MSP is determined by the condition of that the magnetosphere radius equals the star radius, which is proportionally related to the accretion rate as  $\dot{M}^{1/2}$  (White & Zhang 1997). The bottom field of Zsource (Eddington accretion rate) is about  $\sim 10^8$  Gauss, and the bottom field of Atoll-source (1% Eddington accretion rate) will correspond to  $\sim 10^7$  Gauss. The so low field  $\sim 10^7$  Gauss MSPs have not yet been discovered from the radio observation. The final state of NS is constrained by the system parameters, such as star radius, star mass and mass accretion rate, which is nothing to do with the initial field and initial period, so this is why the almost homogeneous field distributions of MSPs from  $\sim 10^8$  (G) to  $\sim 10^9$  (G) do not follow the field distributions of the normal PSRs from  $\sim 10^{11}$  (G) to  $\sim 10^{15}$  (G).

## References

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